

## **FIN 411 -- Investments: Futures Pricing**

**Futures contract is an agreement to buy**

- a fixed amount (& quality) of a product
- at a specified price
- at a specified time in the future

**At the time the contract is begun, no money changes hands (no investment)**

- the value of the contract is returned to zero every day by "settling up"
  - transferring money from the seller(buyer) to the buyer(seller) if the futures price goes up(down)

## **Futures & Forward Contracts**

**Forward contract is an agreement to buy**

- a fixed amount (& quality) of a product
- at a specified price
- at a specified time in the future
- no daily settling up

**Thus, forwards and futures only differ because of the daily settling up provision**

- important to reduce default risk
- randomness in short-term interest rates causes differences in forward & future values

## Financial Futures Contracts: Stocks

### Futures contract on the S&P 500 index:

- Chicago Mercantile Exchange (MERC)
- \$500 times the index value
- very actively traded
- used for hedging large stock portfolios
  - portfolio insurance

## S&P Index Futures: Arbitrage Pricing

### Alternative strategies:

- (1) buy S&P futures at a price  $F_0$  & Treasury bills with an interest rate of  $r_f$  equivalent to buying the stock  $S_0$ 
  - if the price of the index at maturity is  $S_T$ , then the payoff to (1) is:

$$[ S_T - F_0 ] + [ (1+r_f) S_0 ]$$

- (2) buy S&P index at a price  $S_0$  and receive dividend  $D$ 
  - the payoff to strategy (2) is:  
[  $S_T + D$  ]

## S&P Index Futures: Arbitrage Pricing

### Alternative strategies:

- Equating the payoffs to these two strategies gives:

$$F_0 = (1+r_f) S_0 - D$$

- in pseudo-return form:

$$[ F_0 - S_0 ] / S_0 = r_f - d$$

- the percent basis equals the difference between the the interest rate ( $r_f$ ) and the dividend yield ( $d$ ) on the index

## S&P Index Futures: Arbitrage Pricing

Arbitrage relation is sometimes called the "cost-of-carry"

- buying the stock today costs you the time value of money
- but buying the futures contract costs you the dividend that the stockholder receives

## Financial Futures Contracts: Bonds

### Futures contract on 91 day Treasury Bills:

- Chicago Mercantile Exchange (MERC-IMM)
- \$1,000,000 face value
- very actively traded

### Futures contract on 15 year Treasury Bonds:

- Chicago Board of Trade (CBT)
- \$100,000 face value
- 8% coupon assumed

## Tbill Futures Contracts: Arbitrage Pricing

### Forward Interest rates:

- the annualized yield on a k-day Treasury bill at time t is:

$$y(k,t) = \ln[\$1,000/P(k,t)]$$

where  $P(k,t)$  is the price

- buy a 182 day Tbill with a yield  $y(182,t)$  and
- sell a 91 day Tbill with a yield of  $y(91,t)$
- this creates a forward contract in a 91-day Tbill, with a yield:

$$y(91,t+91) = \{ [1+y(182,t)]^2 / [1+y(91,t)]^1 \} - 1$$

## **Tbill Futures Contracts: Arbitrage Pricing**

### **Alternative strategies:**

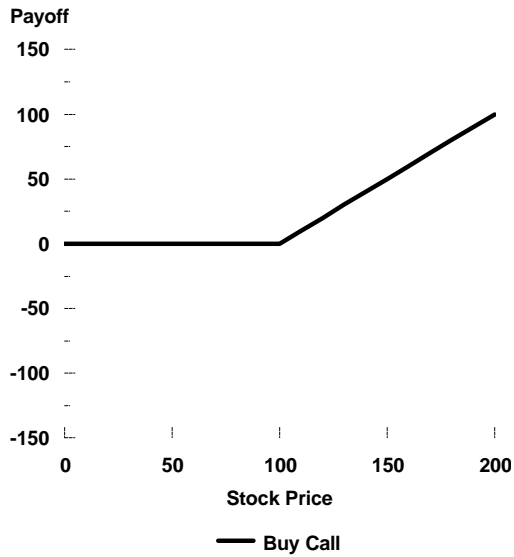
- **buy forwards(futures) if forward rate is higher(lower) than the futures yield**
- **short-sell the other contract**
- **except for variation in interest rates over the life of the contracts (due to "marking to market" -- daily settling up of futures contracts), these are off-setting positions**
  - **(almost) riskless arbitrage**

## **Tbond Futures Contracts: Arbitrage Pricing**

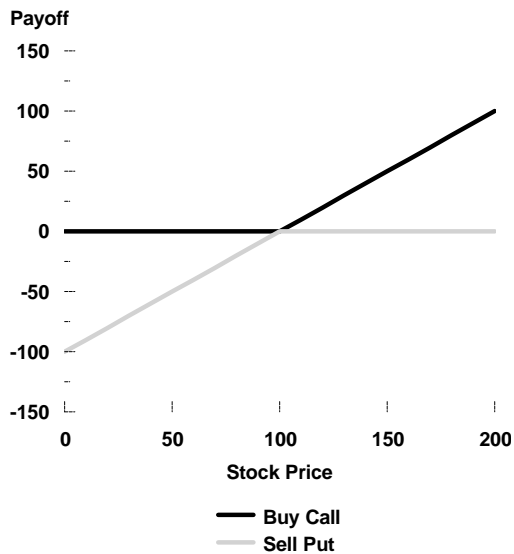
### **Alternative strategies (similar to Tbill analysis of forwards & futures, except):**

- **sellers of Tbond futures have several "quality delivery" options available for delivering of Tbonds when the futures contract matures:**
  - **(a) Which of several securities do you deliver? (about 40 bonds with 15 years to maturity or first call are available)**
  - **(b) Which day in the month do you deliver? (7 days to choose from)**
  - **(c) "Wild-card" option: 6 hours of bond trading after futures delivery price is set**

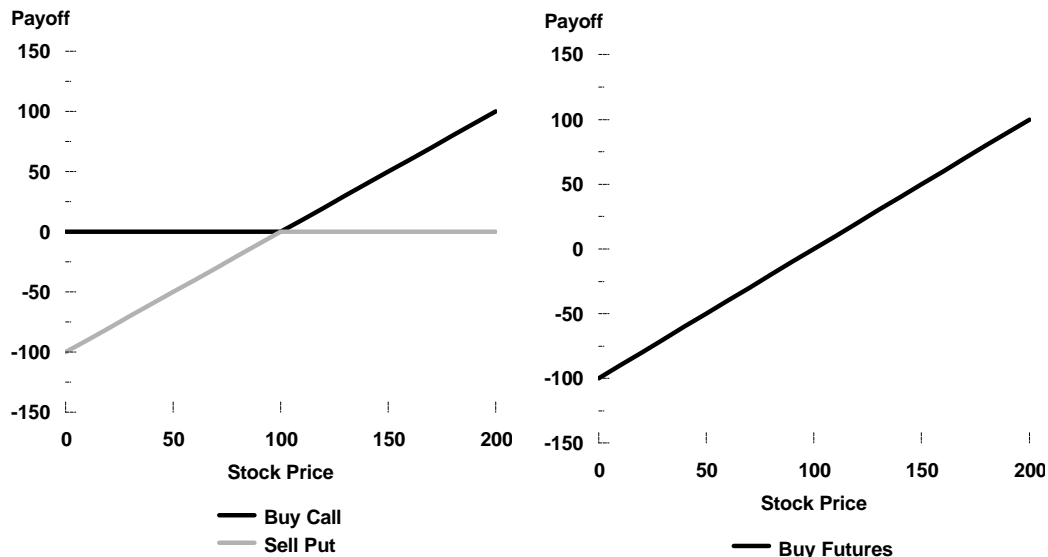
## Financial Futures Contracts: Relation to Option Pricing



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## Financial Futures Contracts: Relation to Option Pricing

**Long futures positions is equivalent to buying a call and selling a put with exercise prices equal to the futures price**

- **selling the futures is equivalent to selling a call & buying a put**

**Payoff from futures contract is symmetric**

- **you can make a lot of money if you guess right**
- **but you can lose a lot of money if you are wrong**

## **Financial Futures Contracts: Other Contracts**

### **Financial:**

- other interest rates (GNMA, LIBOR, Eurodollar, etc.)
- other stock indexes (Value Line, MMI, etc.)
- Foreign exchange
- Metals (gold, silver, etc.)
- Options on Futures

### **Agricultural:**

- corn, wheat, soybeans, potatoes
- pork bellies, cattle
- orange juice, coffee, sugar

## **Financial Futures Contracts: Summary**

**(1) Financial futures are a cheap way to take on a lot of risk**

- low transactions costs
- large leverage (margins of 5 to 15%, versus 50% for stock purchases)
  - i.e., you have to provide about \$100,000 of Treasury bills as collateral to bet \$1,000,000 on stock or bond price moves

**(2) Unless you have (macroeconomic) "inside information," you should use futures (& options) for risk management**

- hedging

## **Hedging with Financial Futures: Portfolio Insurance**

**(3) Used to reduce the (market) risk of a large, well-diversified stock portfolio**

- **(a) buy put options on S&P 500 (or 100) index**
  - puts lower bound on losses
  
- **(b) sell call options on S&P 500 (or 100) index**
  - increases value if stock prices fall or stay level, but you lose if stock prices rise a lot
  
- **(c) sell S&P 500 futures contracts against a portion of your portfolio**
  - as if you were investing in Tbills with that portion of your investment

## **Financial Futures: Questions**

**(1) If you had inside information about a specific company, how might you use options or futures to augment your investment strategy? Discuss:**

- options on individual stocks
- options on market indexes
- futures on market indexes

**(2) If you wanted to adjust the risk of your company's pension fund portfolio, would you use options & futures to do this? Why, or why not?**