The Random Walk Model

Assume the logarithm of 'with dividend' price, ln P(t), changes by random amounts through time:

- $\ln P(t) = \ln P(t-1) + \mu + \varepsilon(it)$ (1)

where:

- P(t) is the sum of the price plus dividend payments made in period t,
- $\mu = E\{\ln[P(t)/P(t-1)]\}$ is the expected continuously compounded return, and
- $\varepsilon(it)$ is the random change in the stock price from period t-1 to period t

Random Walks vs Random Variables

If changes in (log) stock prices are random, then (log) price levels follow a random walk

Distinction between random variables and random walks is confusing for many (most) students

Simple rate of return to an asset is:

- $R(t) = [P(t) - P(t-1) + D(t)] / P(t-1)$ (2)

which is close to the continuously compounded return r(t) for small values of the returns

Simple rate of return

Equation (1) can be rewritten in terms of returns:

- $r(t) = \mu + \varepsilon(it)$ (3)

where the 'unexpected' return in period t is the error term $\varepsilon(it)$

As with the errors from regression models (e.g., APS 402), the errors should be random

Some Properties of the Random Walk Model

The mean and variance of returns are proportional to the length of the measurement interval k

- $E \ln[P(t) / P(t-k)] = k\,\mu$ (4a)
- $\mathrm{Var} \ln[P(t) / P(t-k)] = k\,\sigma^2$ (4b)

More Properties of the Random Walk Model

Autocorrelation tests on returns are equivalent to testing whether the errors are indeed random

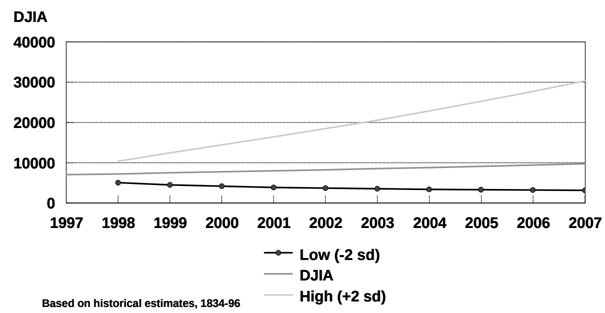
- corr[R(t), R(t-k)] is the autocorrelation coefficient at lag k

Most evidence finds that common stock returns have very small autocorrelations for daily or monthly data for many lags k

- thus, these tests are consistent with the random walk model for stock prices

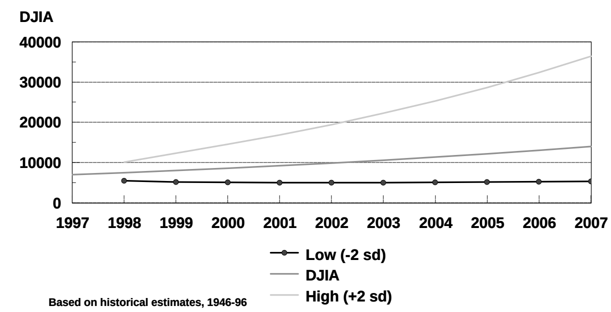
Prediction of Dow Jones Industrial Average

(Random Walk Model, 1997 = 7000)
[Mean = 3.0%, Std Dev = 18.0%]

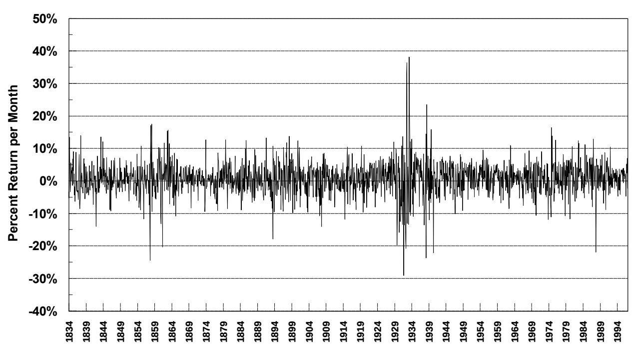


Prediction of Dow Jones Industrial Average

(Random Walk Model, 1997=7000)
[Mean = 7.0%, Std Dev = 15.1%]

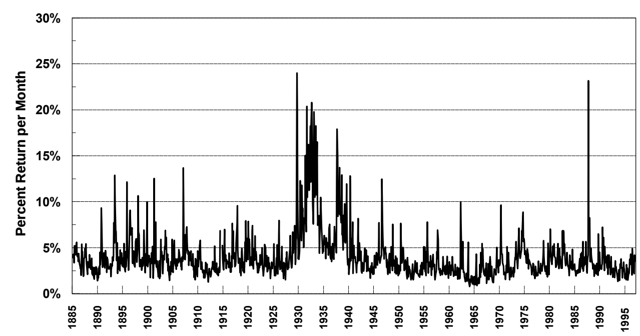


U.S. Monthly Stock Returns, 1834-1996

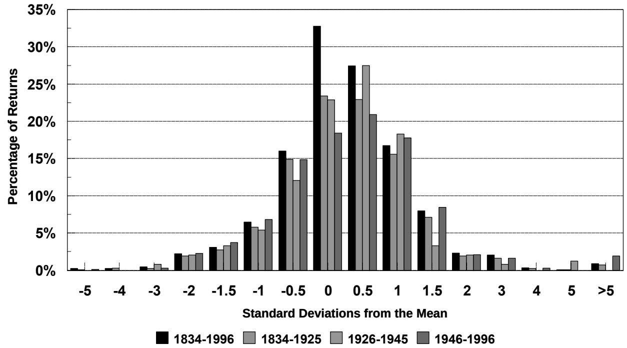


Volatility of Monthly US Stock Returns, 1885-1996

(Based on Daily Returns Within the Month)



Histograms of Monthly US Stock Returns, 1834-1996



Distribution of Stock Returns

A statistic which measures the 'fat-tailedness' of a sample is the <u>studentized range, SR</u>:

- **SR = (Max - Min) / Standard Deviation**

<u>Jarque-Bera statistic (in Eviews)</u> measures whether the skewness & kurtosis statistics are consistent with a Normal distribution

- **skewness should equal 0 and**
- **kurtosis should equal 3**

Distribution of Stock and Bond Returns and Short-term Interest Rates, 1834-1995

	Stock Returns	Long-term Bond Returns	Short-term Yields
Avg	0.79%	0.41%	0.37%
Std	4.89%	1.30%	0.24%
Max	38.28%	8.89%	1.67%
Min	-29.00%	-7.35%	-0.02%
SR	13.75	12.47	7.10
T-stat	7.17	12.77	68.08

Distribution of Stock and Bond Returns and Short-term Interest Rates, 1834 - 1925

	Stock Returns	Long-term Bond Returns	Short-term Yields
Avg	0.67%	0.46%	0.42%
Std	4.36%	1.04%	0.20%
Max	17.59%	6.92%	1.67%
Min	-24.37%	-7.35%	0.13%
SR	9.62	13.75	7.54
T-stat	5.08	12.87	68.78

Distribution of Stock and Bond Returns and Short-term Interest Rates, 1926 - 1945

	Stock Returns	Long-term Bond Returns	Short-term Yields
Avg	0.85%	0.29%	0.09%
Std	8.06%	1.32%	0.13%
Max	38.28%	5.06%	0.43%
Min	-29.00%	-7.83%	-0.02%
SR	8.35	9.75	3.56
T-stat	1.63	3.44	10.93

Distribution of Stock and Bond Returns and Short-term Interest Rates, 1946 - 1995

	Stock Returns	Long-term Bond Returns	Short-term Yields
Avg	1.01%	0.48%	0.38%
Std	4.08%	1.72%	0.26%
Max	16.53%	9.84%	1.35%
Min	-21.81%	-7.20%	0.03%
SR	9.40	9.90	5.18
T-stat	6.07	6.90	34.83

Returns Over More than One Period

When analyzing returns over more the one period it is convenient to use continuously compounded returns,

- **$r(t) = \ln [1+R(t)]$,**

since these returns add up over time

Returns Over More than One Period

Thus, the k-period return $[P(t+k)/P(t)]-1$, is just

- **$[1+R(t+1)][1+R(t+2)]...[1+R(t+k)] - 1$**

  $= R(t+1) + R(t+2) + ... + R(t+k) +$ cross-product terms

These cross-product terms can be important, however

Returns to Different Assets

Period	Asset A	Asset B	Asset C
1	.10	.20	.30
2	.10	.00	-.10
Average 2-period Return	.10	.10	.10
Value of $1 Investment in Period 0 at the end of Period 2	$ 1.21	$ 1.20	$ 1.17

Returns Over More than One Period

Assets with the highest standard deviation of simple returns have the lowest terminal value, for a given level of average simple returns

Returns for More than One Security (i.e., a Portfolio)

Simple returns are easiest to use when measuring the returns to many different securities at the same point in time

Returns for More than One Security (i.e., a Portfolio)

Define a portfolio return as the weighted average of the returns to the N securities in the portfolio:

- R(pt) = Σ w(it) R(it)

where Σ w(it) = 1.

The portfolio weights w(it) represent the proportion of wealth invested in asset i at the beginning of period t.

Returns for More than One Security (i.e., a Portfolio)

If an investor put equal dollar amounts in each of N securities, this would be an equal-weighted portfolio

- w(it) = w(i) = 1 / N

If one invested in proportion to the outstanding market value (i.e., price times shares outstanding) of each of N securities, this would be a value-weighted portfolio

- w(it) = value of asset i / total value of all N assets

Returns for More than One Security (i.e., a Portfolio)

The return to an equal-weighted portfolio is the average return to the assets in the portfolio in period t, so it is easy to compute

- but it is hard to maintain an equal-weighted portfolio through time
- you must rebalance every period as the value of the holdings change

Returns for More than One Security (i.e., a Portfolio)

The return to a value-weighted portfolio is difficult to compute because the value-weights change every period

- but it is easy to maintain a value-weighted portfolio
- it requires no rebalancing (except to account for new issues or retirements of securities)

Returns for More than One Security (i.e., a Portfolio)

The S&P 500 and the CRSP value-weighted portfolios are examples of value-weighted portfolios

- By construction, a value-weighted portfolio places larger weight on large firms
- therefore smaller weight on small firms relative to an equal-weighted portfolio

Returns for More than One Security (i.e., a Portfolio)

A comparison of the returns to the CRSP value- and equal-weighted portfolios of all NYSE stocks from 1926-95 shows this effect

- for 1926-95, the mean monthly returns are .96% and 1.31% for the value and equal-weighted portfolios
- the monthly standard deviations of returns are 5.53% and 7.61%

Thus, the risks and returns to small stocks are probably higher than for large stocks.

Distribution of CRSP Value & Equal-weighted Stock Returns, 1926-1995

	Cont Comp CRSP VW Returns	Simple CRSP VW Returns	Simple CRSP EW Returns
Avg	0.80%	0.96%	1.31%
Std	5.52%	5.53%	7.61%
Max	32.41%	38.28%	65.51%
Min	-34.25%	-29.00%	-31.23%
SR	12.08	12.16	12.71
t-test	4.22	5.03	4.98

Distribution of CRSP Value & Equal-weighted Stock Returns, 1926-1945

	Cont Comp CRSP VW Returns	Simple CRSP VW Returns	Simple CRSP EW Returns
Avg	0.53%	0.85%	1.64%
Std	7.99%	8.04%	11.66%
Max	32.41%	38.28%	65.51%
Min	-34.25%	-29.00%	-31.23%
SR	8.34	8.37	8.29
t-test	1.02	1.63	2.18

Distribution of CRSP Value & Equal-weighted Stock Returns, 1946-1995

	Cont Comp CRSP VW Returns	Simple CRSP VW Returns	Simple CRSP EW Returns
Avg	0.91%	1.00%	1.18%
Std	4.13%	4.12%	5.16%
Max	15.33%	16.56%	29.92%
Min	-25.48%	-22.49%	-27.10%
SR	9.89	9.48	11.05
t-test	5.43	5.97	5.58

Market Model Regressions

Market model regression:

- $R(it) = \alpha(i) + \beta(i) R(mt) + \varepsilon(it)$, t = 1,...,T (5)

where R(it) is the return to asset i and R(mt) is the return to the "market" portfolio of assets in period t is a time series regression model

- the slope coefficient $\beta(i)$ ("beta") is a measure of the relative nondiversifiable risk of the asset which is the dependent variable in the regression [R(it)] as part of the portfolio which is the independent variable [R(mt)]

Market Model Regressions

Weighted average beta $\Sigma\,\beta(i)$ equals 1 by construction for the set of assets i which make up the regressor portfolio [R(mt)]

Weighted average intercept $\Sigma\,\alpha(i)$ in the market model regression (5) must equal 0 by construction

Market Model Regressions

If the Capital Asset Pricing Model (CAPM) is true, then the intercept

- $\alpha(i) = [1-\beta(i)]\, R(f)$

where R(f) is the risk-free rate of return

- so betas $\beta(i) > 1$ would typically be associated with $\alpha(i) < 0$

Market Model Regressions

Table below shows the market model regression using the CRSP value-weighted portfolio as the regressor,

- $R(et) = \alpha + \beta\, R(vt) + \varepsilon(it)$

where R(et) is the simple return to the equal-weighted portfolio and R(vt) is the simple return to the value-weighted portfolio in period t

Market Model: Equal-weighted Returns on Value-weighted Returns

	1926-95	1926-45	1946-95
α	0.10%	0.49%	0.08%
β	1.257	1.365	1.095
$t(\beta=1)$	5.69	5.71	2.57
R^2	.835	.886	.762

Risk Premium Market Model: CAPM Test for the Equal-weighted CRSP Portfolio

	1926-95	1926-45	1946-95
α	0.18%	0.52%	0.11%
$t(\alpha=0)$	1.86	2.23	1.12
β	1.258	1.366	1.094
$t(\beta=1)$	5.72	5.73	2.58