FIN 411 -- Testing the CAPM

We will want to look at average returns realized by assets with different levels of risk [β(i)], then see whether average returns are linearly related to risk

(1) need to get as much dispersion in "true" betas as possible

(2) need to have as little estimation error in estimates of expected returns & betas as possible

FIN 411 -- Testing the CAPM

<u>Sharpe-Lintner CAPM (riskfree asset):</u>

$$E[(R(i)] = R(f) + \{E[R(m)] - R(f)\}\ \beta(i)$$

where $\beta(i) = cov[R(i),R(m)]/\sigma^2[R(m)]$ is the relative marginal contribution of asset i to the risk of the market portfolio m

More general versions of the CAPM predict a linear relation between expected returns & betas, but the intercept doesn't equal the risk-free rate

Testing the CAPM: Sharpe-Cooper (<u>FAJ</u>,1972)

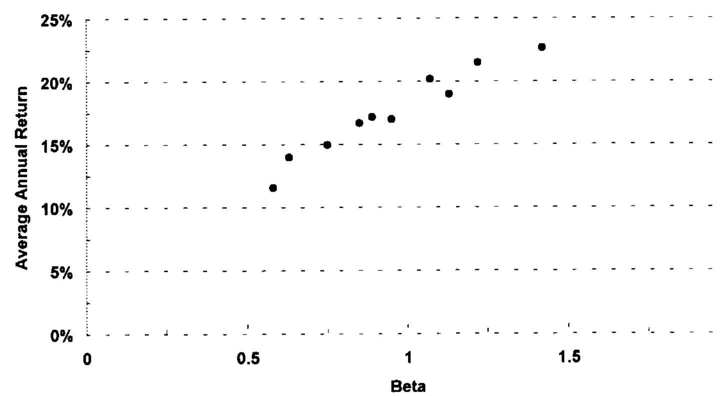
Compute avg annual returns and betas for 10 portfolios sorted by beta values estimated from a prior 5 year period

(1) equal-weighted portfolios of NYSE stocks

(2) strong positive relation between risk & return:

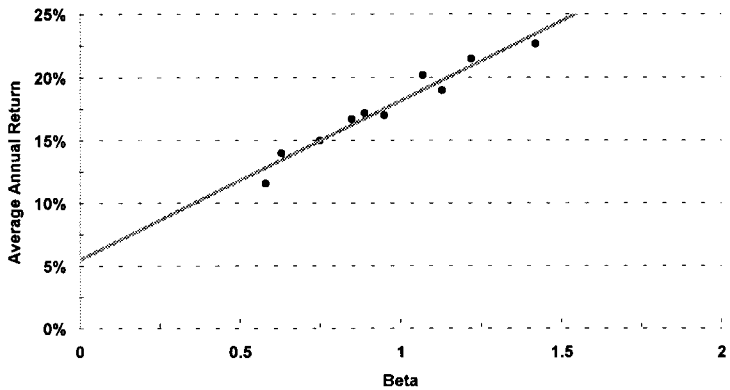
slope, or risk premium = 12.75%;
intercept, or 'zero-beta' return = 5.54%

Testing the CAPM: Sharpe-Cooper (<u>FAJ</u>,1972)



1931-67: a = .0554, b=.1275

Testing the CAPM: Sharpe-Cooper (FAJ,1972)



1931-67: a = .0554, b=.1275

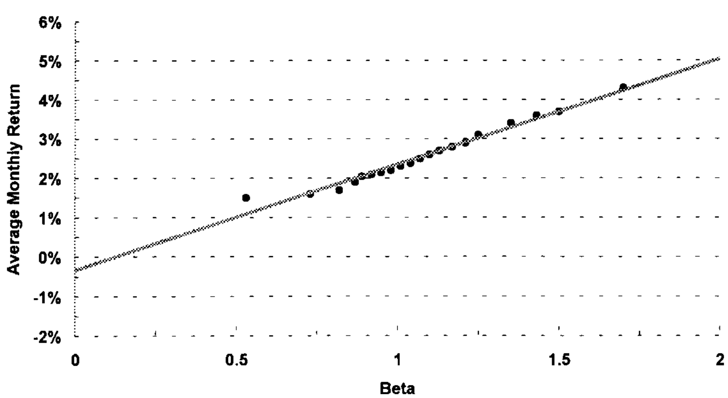
Testing the CAPM: Black-Jensen-Scholes (1972)

Compute avg monthly returns and betas for portfolios sorted by beta values estimated from a prior period, then estimate cross-sectional regression for 8 year sample period

(1) equal-weighted portfolios of NYSE stocks

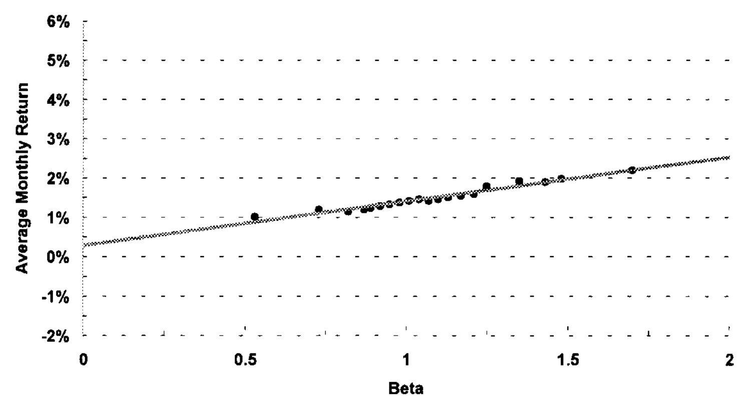
(2) strong positive relation between risk & return in 1931-39, smaller in 1939-47, and actually negative (but small) from 1957-65

Black-Jensen-Scholes: 1931-39



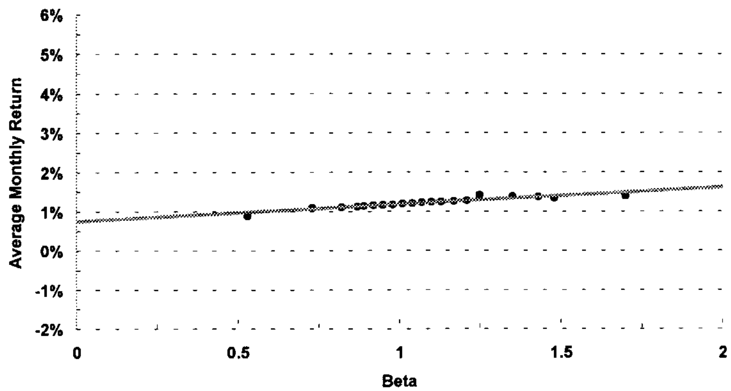
a=-.003, b=.027

Black-Jensen-Scholes: 1939-47



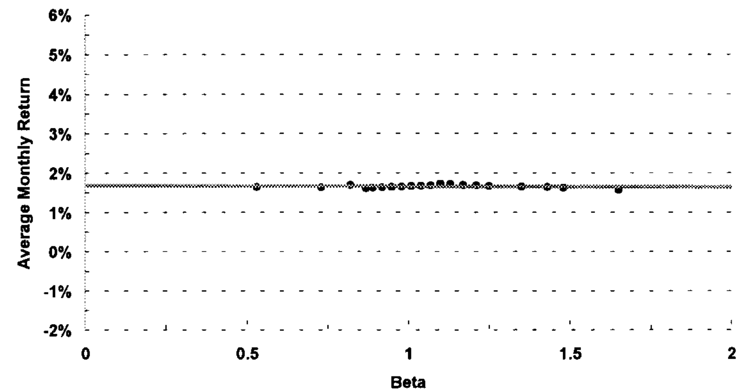
a=.0029, b=.011

Black-Jensen-Scholes: 1948-57



a=.0075, b=.0043

Black-Jensen-Scholes: 1957-65



a=.0169, b=-.0002

Testing the CAPM: Fama-MacBeth (1973)

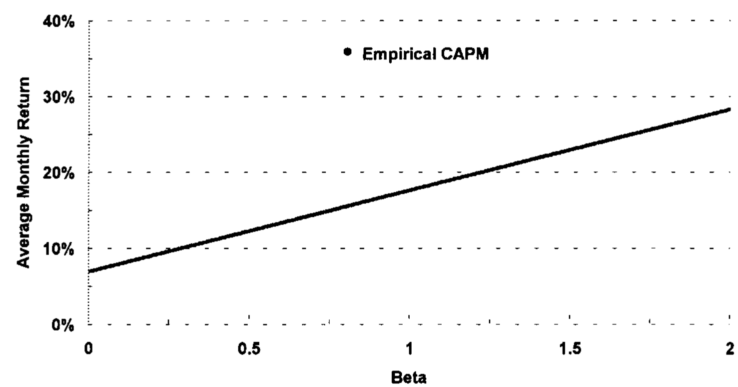
Estimate cross-sectional regression of returns vs. betas for portfolios sorted by beta values estimated from a prior period, then average the estimates of the risk premium (slope) and the risk-free rate (intercept)

(1) 20 equal-weighted portfolios of NYSE stocks

(2) t-statistic is:

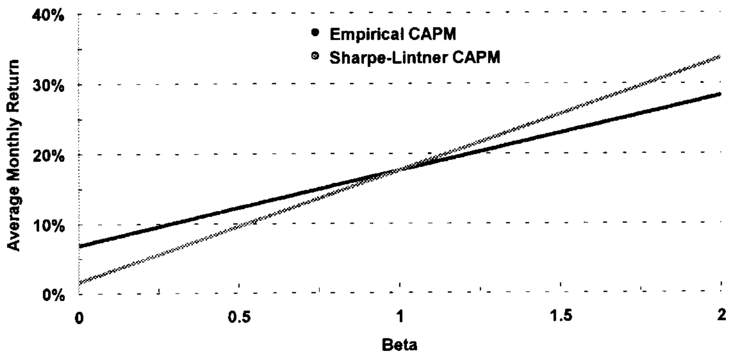
$$[\text{sample average}]/\,[\text{sample variance}/T]^{1/2}$$

Fama-MacBeth: 1938-68



a=.0732, b = .1030

Fama-MacBeth: 1938-68



a=.0732, b = .1030

Fama-MacBeth (1973): <u>Conclusions</u>

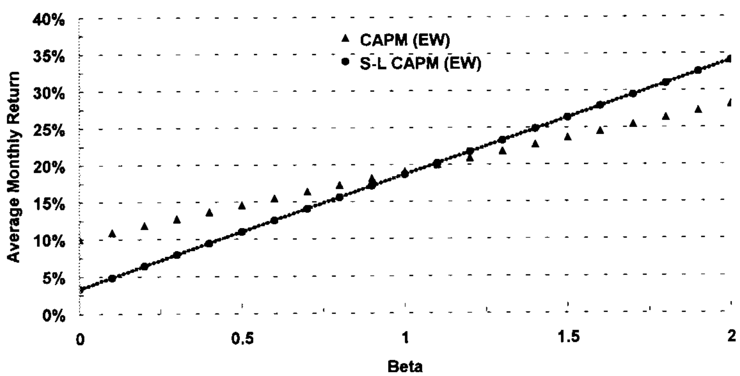
(1) It looks like average returns and betas are (reasonably) linearly related

(2) The expected return on a portfolio with a beta of zero is higher than the return on one-month Tbills R(f)

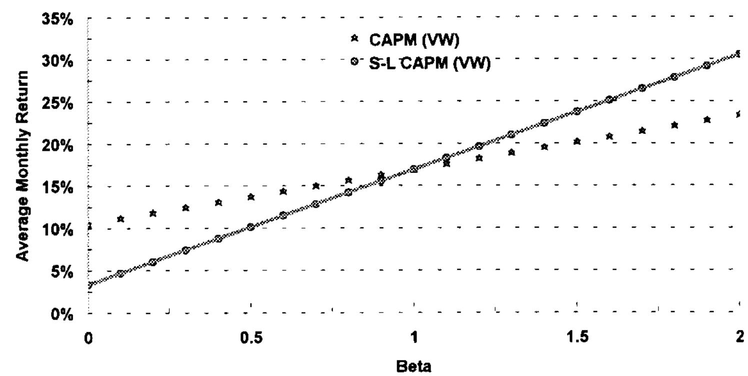
(3) Therefore, the risk premium (slope) is smaller than the Sharpe-Lintner model would predict,

$$\{E[R(m)] - R(f)\} > \{E[R(m)] - E[R(z)]\}$$

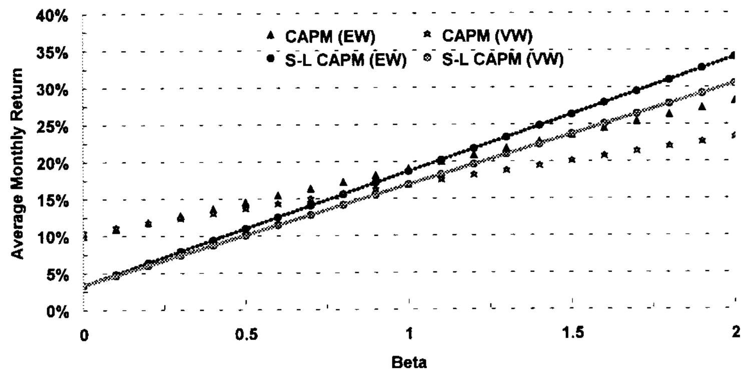
Replication of Fama-MacBeth: 1931-90



Replication of Fama-MacBeth: 1931-90



Replication of Fama-MacBeth: 1931-90



What Happens with Different Market Portfolios?

(1) Replicate Fama-MacBeth tests using 5 size-ranked portfolios {equity capitalization: shares outstanding times price at the beginning of the year} and 12 industry portfolios {SIC codes}

(2) Use both the CRSP equal-weighted (EW) and value-weighted (VW) portfolios of NYSE stocks as market portfolios

(3) Risk premium is larger for VW betas, but zero-beta expected return is too high for both market portfolios

Errors-in-Variables Problems in Linear Regression

Suppose the "true" regression model is:

$$Y(i) = \alpha(i) + \beta(i)\, X(i) + \varepsilon(i)$$

but you can only observe estimates of Y(i) and X(i),

$$y(i) = Y(i) + u(i)$$
$$x(i) = X(i) + v(i)$$

The regression of y(i) on x(i) yields biased estimates of the "true" regression parameters $\alpha(i)$ and $\beta(i)$, even though the estimation errors u(i) and v(i) are random and have mean zero

Errors-in-Variables Causes Biased Regression Parameter Estimates

$$y(i) = a(i) + b(i)\, x(i) + e(i)$$

The least squares estimate of b(i) is biased towards zero (and a(i) is biased upwards if $\beta(i) > 0$ and $avg[X(i)] > 0$):

$$b(i) = cov[y(i),x(i)]/var[x(i)]$$
$$= cov[Y(i),X(i)]/\{var[X(i)] + var[v(i)]\}$$
$$< \beta(i) = cov[Y(i),X(i)]/var[X(i)]$$
$$a(i) = avg[y(i)] - b(i)\, avg[x(i)]$$
$$= avg[Y(i)] - b(i)\, avg[X(i)]$$
$$> \alpha(i) = avg[Y(i)] - \beta(i)\, avg[X(i)]$$

if $\beta(i) > 0$ and $avg[X(i)] > 0$

Errors-in-Variables Problems in CAPM Tests

Suppose the "true" regression model is:

$R(it) = \gamma(0t) + \gamma(1t)\ \beta(it) + u(it), \quad i = 1,...20$

where $\gamma(1t)$ estimates the risk premium and $\gamma(0t)$ estimates the expected return to a portfolio with a beta of 0

- estimation errors in average portfolio returns R(it) simply add more noise to the error term u(it), not bias to the regression
- estimation errors in betas $\beta(it)$ causes the risk premium $\gamma(1t)$ to be biased down and the zero-beta return $\gamma(0t)$ to be biased up

Effects of Estimation Error on CAPM Tests

(1) Simulate Fama-MacBeth portfolio returns with different assumptions about estimation error vs. dispersion of true betas

(2) Std Error of <u>estimates</u> of average returns is 1.4% per year in both cases

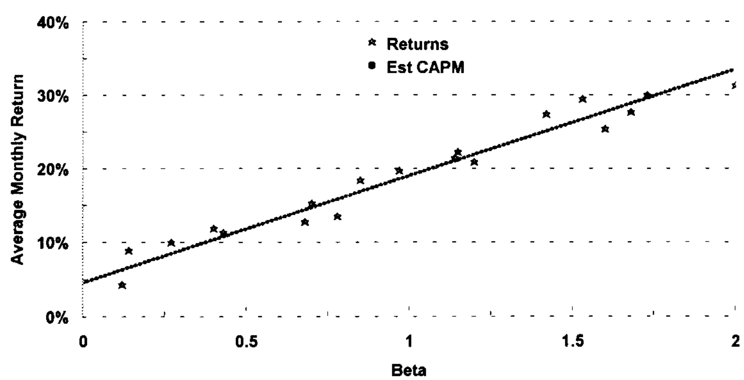
- [i.e., estimation error in expected returns]

(3) Std Error of <u>estimates</u> of beta is either .06 or .6

(4) True betas are either uniform from 0 to 2, or all equal to 1

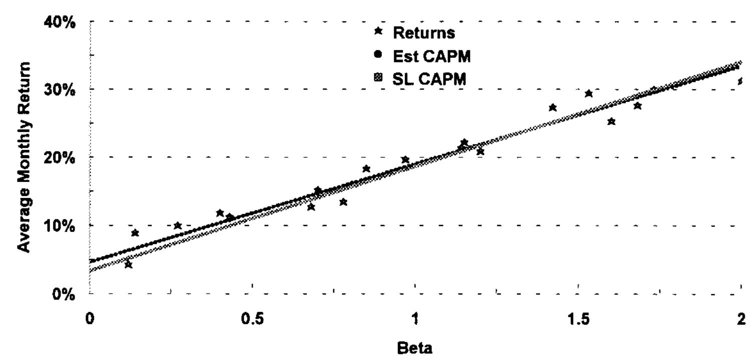
- set so the estimated betas range from about 0 to 2

Effects of Small Estimation Errors on CAPM Tests



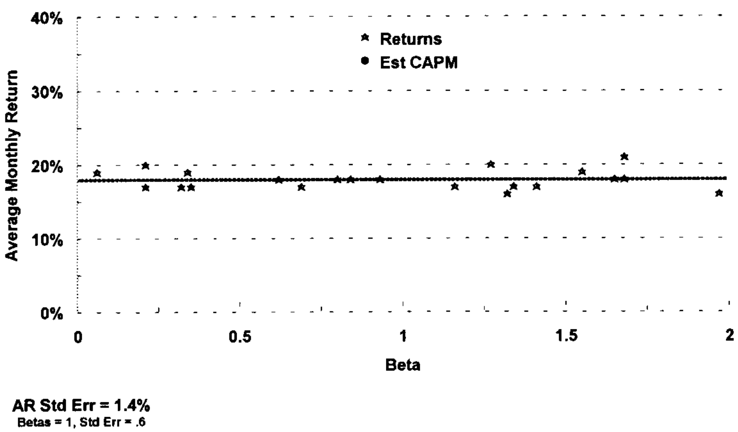
AR Std Err = 1.4%
Betas = {0, 2}, Std Err = .06

Effects of Small Estimation Errors on CAPM Tests

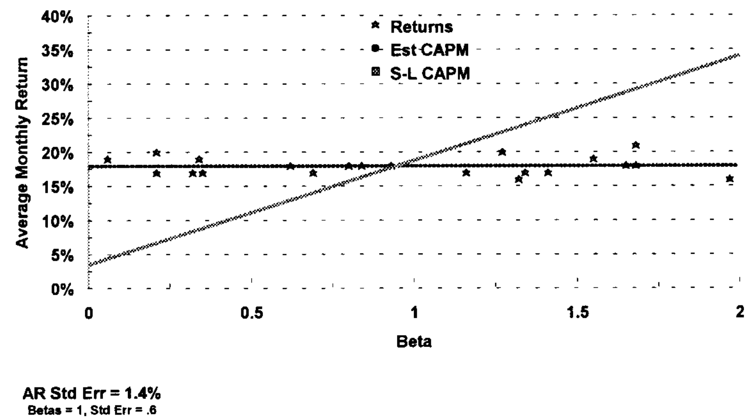


AR Std Err = 1.4%
Betas = {0, 2}, Std Err = .06

Effects of Large Estimation Errors on CAPM Tests



Effects of Large Estimation Errors on CAPM Tests



Testing the CAPM: Summary

(1) Strong evidence that expected returns increase with risk

(2) Evidence that risk-return relation is flatter than Sharpe-Lintner CAPM predicts

- i.e., high beta stocks have lower returns & low beta stocks have higher returns

(3) Estimation error in betas may explain part of this problem

Testing the CAPM: Additional Summary

(4) Even without estimation error, if you use the wrong market portfolio to estimate betas, the slope and intercept of the risk/return trade-off will not coincide with the Sharpe-Lintner model

- If the "market" portfolio you use to estimate risk is mean-variance efficient, but has higher risk than the "true" (value-weighted) market portfolio of all marketable assets, then the expected return on a zero-beta portfolio should be higher than the risk-free rate Rf

Testing the CAPM: Questions

(1) Would you use the theoretical (Sharpe-Lintner) or the empirical CAPM for capital budgeting/performance evaluation? Why?

(2) A pension fund consultant uses the S&P 500 index as a benchmark for the performance of common stock investments. Ibbotson & Sinquefield estimate that the average risk premium for the S&P index is about 8.4% per year. What index should you use to estimate betas if you want to use risk-adjusted performance methods? Why?