TECHNICAL NOTE

Unitary Manufacturing Cell Design with Random Product Feedback Flow

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Abstract: This paper presents a capacity model that incorporates the influence of stochastic feedback flow on the productivity of a unitary manufacturing cell. A Unitary Manufacturing Cell is defined as an automated group of work stations served by a robot, and producing one single product at a time. Process control considerations and the need to rework a certain portion of produced items usually results in recirculation, or feedback flow. A statistical analysis of parts flow with recirculation is presented along with an illustration of the oscillatory characteristic of its total-cell-time probability density function.

■ The system of interest in this paper is called a cellular manufacturing system, also referred to as manufacturing cell or work cell. It is a part of a production line designed to fabricate families of parts having similar geometry and process requirements. Typicaly, its output is completed components. For an overview of cellular manufacturing technology see Black [1], Schonberger [13], and Zisk [19]. In many cases today, a manufacturing cell is comprised of several automatic work statons or machines with an industrial robot [5, 12]. The robot's typical role is to transfer parts from an input conveyor to and between machine tools, and finally to an output conveyor, [see Fig. 1]. In these robotic cells *automatic inspection* and *immediate feedback* are applied to ensure high quality products [1] and [13].

In this paper we study the *Unitary Manufacturing Cell* and evaluate the nature of its performance. It produces one single product at a time and all parts are examined in an inspection station which is in the cell, and if necessary are recycled and reworked until they pass inspection. A common reason for unitary cell design is to avoid frequent changes in tooling and accessories, e.g., paint color, on a mixed model production facility. Various unitary applications include ceramic mold-making systems for investment casting foundries, [15, pp. 151-163], trimming an injection molded part with the aid of drilling and milling units [16, pp. 231-234], or the polishing of ten different high-grade steel washbasins [16, pp. 235-236]. A recent study discussed the optimal part selection policy for a unitary manufacturing cell [14].

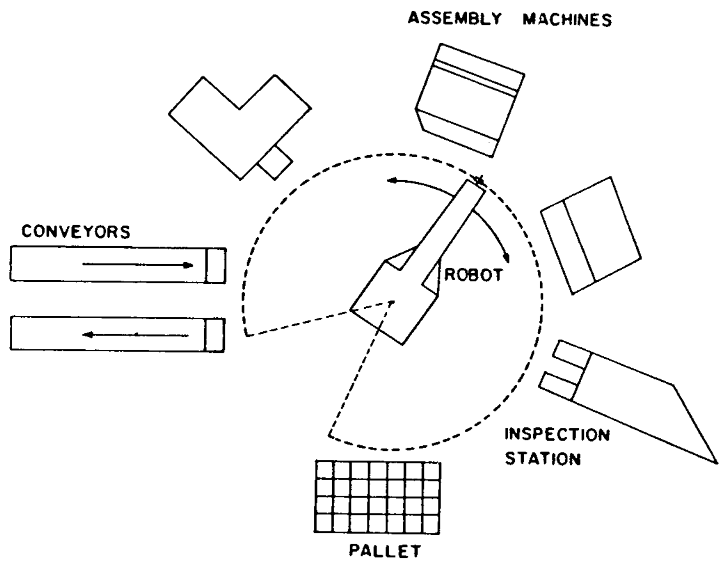

Fig. 1. Physical System configuration of Robotic Assembly Cell.

The objective of this article is to analyze the product recirculation in unitary cells. This is accomplished by direct probabilistic analysis which provides tractable solutions for major operational performance measures. In order to esti-

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mate performance measures, e.g., passage times in the cell, a probabilistic characterization for a typical job is performed. A similar approach has been taken in a number of probabilistic measurements of production systems that were not amenable to queueing analysis [4, p. 336], [14] and [17].

UNITARY MANUFACTURING CELL MODEL

Assume that:

(1) A manufacturing cell is partitioned into two active areas: Area M located in the *main* production line and area R in the *recycle* line, or reworking line.

(2) Parts arrive singly on an input conveyor to area M.

(3) When a part has completed a tour through area M, it is also inspected. It will leave the cell if it passes inspection. Otherwise, it will be recirculated for rework at R and then return to M.

(4) Until a successfully finished part (product, or production order) leaves the cell, no other part can enter ("unitary operation").

(5) Although scrap parts are not modeled directly, scrap is allowed and faulty parts are replaced within the cell, as a part of the rework operation.

(6) Inspection may mean the actual measurements, gauging, or testing of a part. However, in broad terms inspection may mean any identification of faulty conditions that necessitate additional processing of the part. For example, if an integrated circuit chip is found defective during the inexpection, it will automatically be replaced by another chip. The replacement operation is considered as part of the rework. Thus, an *actual inspection* or *gauging* could be considered as part of the *main* area, whereas the *results* of the inspection will be considered *rework* if special additional actions have to be taken

The probability of an individual part to pass inspection and thus leave the cell is q; the probability to remain for rework is $p = 1 - 1$. Figure 2 illustrates the general flow of parts in the system. Let S_M and S_R be the actual time for working one part *once* at M ("Manufacturing time") or at R ("rework time"). Denote the probability density function of S_M and S_R by $h_M(t)$ and $h_R(t)$, respectively.

The choice of $h_M(t)$ and $h_R(t)$ can be based upon some theoretical considerations of the underlying manufacturing process, or it can be estimated statistically from historical operational data [3], [10]. It follows that S_M and S_R are independent random variables and the distribution of time in each area is the same for every part. This is true when the rework time is the same even if the part has already been reworked once, or several times. It happens, for instance, in various assembly systems where the rework station takes out and realigns (or replaces) the defective items. Once this has been done, the assembly operation is attempted again in the main area. In these cases the probability of rerouting is independent of the number of times that the part has already passed through the main area.

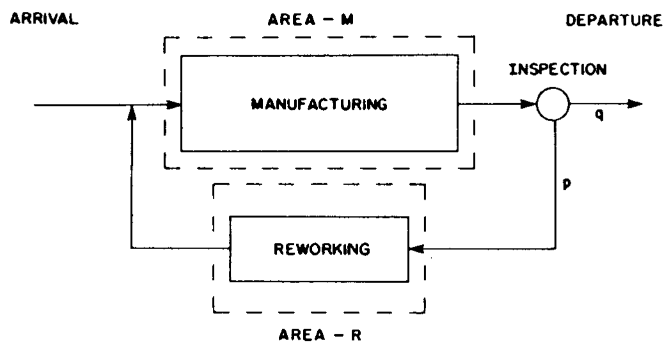

Fig. 2. Feedback flow in manufacturing cell.

Further, denoted by μ_M, σ_M^2, μ_P^2 and σ_R^2 the means and variances of S_M and S_R. Define by $m_M(x)$ and $m_R(x)$ the moment generating functions of M and R. These functions always exist in practice since the activity times are strictly bounded.

The total time a product spends at the cell depends on the distribution of production times, as well as on the number of cycles completed. In order to characterize the state of a part, we let N denote the number of times a part passes through area M. By definition, N is a discrete and positive random variable whose p.d.f. $k(n)$ is the real valued function:

$$k(n) = \Pr[N = n] = qp^{n-1} \quad n = 1, 2, 3 \ldots \quad (1)$$

The corresponding distribution function is given by

$$K(n) = \Pr[N \leqslant n] = 1 - p^n \quad n = 1, 2, 3, \ldots \quad (2)$$

N has a geometric distribution function with mean $\mu_N = 1/q$ and variance $\sigma_N^2 = p/q^2$.

Total Cell Time

Let the continuous random variable θ denote the actual total time a part spends at the cell. Define $h_\theta(t)$ as the probability density function of θ where $h_\theta(t) > 0$, $t \in S$ and $S = \{t: 0 < t < \infty\}$.

The conditional probability density for θ, given that exactly n cycles took place, is denoted by (t/n). It is intuitively clear that

$$\pi(t/1) = h_M(t)\,. \tag{3}$$

For $n = 2$, the conditional probability is

$$\pi(t/2) = [h_M \cdot h_R \cdot h_M])(t) \tag{4}$$

where the convolution of two functions f and g is denoted by $f * g$. Since the convolution operator is commutative and associative [7] we can write

$$\pi(t/2) = [h_M^{*(2)} \cdot h_R](t)\,. \tag{5}$$

where $f^{*(n)}$ denotes the n-fold convolution of f with itself.

The density of θ is obtained by conditioning and then unconditioning on the random number of cycles, n. Thus,

$$h_\theta(t) = \sum_{n=0}^{\infty} k(n+1)\,\pi(t/n)$$

$$= q h_M(t) + g \sum_{n=2}^{\infty} p^{n-1} [h_M^{*(n)} \cdot h_R^{*(n-1)}](t)\,. \tag{6}$$

In general, this function is not convenient for direct numerical evaluation. However, given closed form expressions for the moment generating function of $h_\theta(t)$, it can be computed using standard algorithms [2] for inverting Laplace transforms (see Fig. 3).

Define by $m_\theta(x)$ the moment generating function of θ and recall that the moment generating function of the sum of independent variables is equal to the product of their respective moment generating functions. Using the fact that the expectation is a linear operator we can interchange summation and integration in order to show that, if $m_\theta(x)$ exists, it can be written as

$$m_\theta(x) = q m_M(x) + q p m_M^2(x) m_R(x) + \ldots$$

The moment generating function, when it exists, uniquely characterizes the distribution.

The mean μ_θ and the variance σ_θ^2 of θ can be computed directly from $m_\theta(x)$. Alternatively, μ_θ and σ_θ^2 are computed here using the well known distribution independent results for the mean and variance of the random sums of random variables [6, pp. 286-301]. Since each part passes N times through M and $(N - 1)$ times through R we get

$$\mu_\theta = \mu_M \mu_N + \mu_R(\mu_N - 1)$$

$$= (\mu_M + p\mu_R)/q \tag{8}$$

and

$$\sigma_\theta^2 = (\mu_N - 1)(\sigma_M^2 + \sigma_R^2) + (\mu_M + \mu_R)^2 \sigma_N^2 + \sigma_M^2$$

$$= (\sigma_M^2 + p\sigma_R^2)/q + p(\mu_M + \mu_R)^2/q^2\,. \tag{9}$$

The production rate is given by $1/\mu_\theta$. Note that the relative impact of μ_R on the mean production rate is a function of p and it becomes important when rework becomes more likely.

Total Manufacturing and Total Rework Times

In some cases we may be interested in computing the total time that a product or the material handling device stays at each area (areal times). Define by $S_{\bar{M}}$ and $S_{\bar{R}}$ the *total* actual time that an item spends at area M or at R. The total time in system is $\theta = S_{\bar{M}} + S_{\bar{R}}$. Let $h_{\bar{M}}(t)$ and $h_{\bar{R}}(t)$ denote the probablility density function for $S_{\bar{M}}$ and for $S_{\bar{R}}$, respectively; their moment generating functions are $m_{\bar{M}}(x)$ and $m_{\bar{R}}(x)$. The corresponding means and variances of these distributions are $\mu_{\bar{M}}$ and $\sigma_{\bar{M}}^2$, $\mu_{\bar{R}}$ and $\sigma_{\bar{R}}^2$.

The approach for analyzing the areal times is identical to the approach for the total cell time. One can verify, therefore, that

$$h_{\bar{M}}(t) = q \sum_{n=1}^{\infty} p^{n-1} [h_M^{*(n)}]\,(t) \tag{10}$$

$$h_{\bar{R}}(t) = q \sum_{n=1}^{\infty} p^{n-1} [h_R^{*(n-1)}]\,(t) \tag{11}$$

$$m_{\bar{M}}(x) = q\, m_M(x)/[1 - p\, m_M(x)] \tag{12}$$

$$m_{\bar{R}}(x) = q/[1 - p\, m_R(x)] \tag{13}$$

where $h^{*(0)}(t) = \delta(t)$, is the Dirac delta function.

The corresponding means and variances are given by

$$\mu_{\bar{M}} = \mu_M q^{-1} \tag{14}$$

$$\sigma_{\bar{M}}^2 = q^{-1}(pq^{-1}\mu_M^2 + \sigma_M^2) \tag{15}$$

$$\mu_{\bar{R}} = p\,q^{-1}\mu_R \tag{16}$$

$$\sigma_{\bar{R}}^2 = q^{-1}(p\,q^{-1}\mu_R^2 + p\,\sigma_R^2)\,. \tag{17}$$

Since $\mu_\theta = \mu_{\bar{M}} + \mu_{\bar{R}}$, the analysis in this section partitions the total manufacturing cell time into its two major components. This information is useful in order to estimate the *relative weight* that each functional area contributes to the mean cell time. While the expected areal times add up to the total expected cell times it should be noted that $\sigma_\theta^2 \geqslant \sigma_{\bar{M}}^2 + \sigma_{\bar{R}}^2$. The difference in variances, $2pq^{-2}\mu_R\mu_M$, is due to the statistical dependence of the two areal times $S_{\bar{M}}$ and $S_{\bar{R}}$. Therefore, one gets that [6, p.230]

$$\text{Covariance}\,(S_{\bar{M}}, S_{\bar{R}}) = E[S_{\bar{M}} - E(S_{\bar{M}}) \quad S_{\bar{R}} - E(S_{\bar{R}})]$$

$$= 0.05(\sigma_\theta^2 - \sigma_{\bar{R}}^2 - \sigma_{\bar{M}}^2) = pq^{-2}\mu_R\mu_M\,. \tag{18}$$

Table 1: Summary of general measures.		
Measure	Mean Value	Variance Value
(1) No. of visits in area M:	$\mu_N = 1/q$	$\sigma_N^2 = p/q^2$
(2) Total time in area M:	$\mu_{\bar{M}} = \mu_M q^{-1}$	$\sigma_{\bar{M}}^2 = q^{-1}(pq^{-1}\mu_M^2 + \sigma_M^2)$
(3) Total time in area R:	$\mu_{\bar{R}} = \mu_R p q^{-1}$	$\sigma_{\bar{R}}^2 = q^{-1}(pq^{-1}\mu_R^2 + p\sigma_R^2)$
(4) Total time is the cell:	$\mu_\theta = (\mu_M + p\mu_R)q^{-1}$	$\sigma_\theta^2 = (\mu_M + \mu_R)^2 pq^{-2} + (\sigma_M^2 + p\sigma_R^2)q^{-1}$
(5) Covariance between $S_{\bar{M}}, S_{\bar{R}}$:		$\mathrm{COV}(S_{\bar{M}} S_{\bar{R}}) = pq^{-2}\mu_R\mu_M$
(6) Correlation between $S_{\bar{M}}, S_{\bar{R}}$		$\rho(S_{\bar{M}}, S_{\bar{R}}) = \mathrm{COV}(S_{\bar{M}}, S_{\bar{R}})/(\sigma_{\bar{M}}, \sigma_{\bar{R}})$

The correlation coefficient of $S_{\bar{M}}$ and $S_{\bar{R}}$ is

$$\rho(S_{\bar{M}}, S_{\bar{R}}) = \mathrm{cov}(S_{\bar{M}}, S_{\bar{R}})/(\sigma_{\bar{M}}, \sigma_{\bar{R}}) . \tag{19}$$

Several general measures that have been developed above are summarized for easy reference in Table 1.

APPLICATION TO UNITARY MANUFACTURING CELL DESIGN

Consider a unitary assembly machine cell with L stations that performs a series of component mating, screw driving, nut running, and staking operations (see Fig. 1). Base parts arrive at the cell on a conveyor and an industrial robot is used to transfer the parts between the conveyor and the machines. All base parts are inspected before arrival and they are assumed to be of high quality. The main advantages of applying a robot in this unitary cell are the variable routing of parts in the cell coupled with the requirement for accurate part positioning on the assembly machines. Experience with several programmable assembly cells with sensory feedback, variable task times and with rework led to this application [8, 9, 10, 11, 15].

The series of assembly operations terminates at an inspection station. At this station, poor assemblies are rejected with probability p; the robot will strip the faulty items from the base part and palletize them for further manual inspection. The base parts are used for another assembly pass through the main area. Good assemblies are delivered to a subsequent production cell by another, outgoing conveyor.

The actual elapsed times for each assembly operation in the example cell are assumed to be exponentially distributed with mean $1/\mu$. The robot's total travel time with an assembly through the main area in one passage is a constant α time units. This travel time includes the clamping, unclamping, and gauging operations. As a result, the p.d.f. of a single passage through area M is given by the following shifted gamma distribution [7]:

$$h_M(t) = \begin{cases} \exp[-\mu(t-\alpha)]\mu^L (t-\alpha)^{L-1}/(L-1)! & t > \alpha \\ & L = 1,2,3,\ldots \\ 0 & \text{Otherwise} . \end{cases} \tag{20}$$

When an assembly is rejected as a result of the inspection, it is returned for additional processing. In this case the rework at area R consists of the stripping and the palletization of faulty items. We assume that a single passage at R takes β time units. Since S_R is a constant, the determination of $h_R(t)$ involves the use of the unit impulse, or Dirac delta function, $\delta(t)$ [3]. Note that $\delta(t)$ is the derivative of the unit step function $U(t)$. Hence,

$$\delta(t) = \lim_{\epsilon \to 0} \frac{1}{\epsilon} [U(t) - U(t-\epsilon)] \tag{21}$$

$$\int_0^\infty \delta(t)dt = U(t) = 1 . \tag{22}$$

Using the shifting property of $\delta(t)$ we obtain that

$$h_R(t) = \delta(t-\beta) . \tag{23}$$

The density of the total time in the system is given by substituting Equations (20) and (21) into Equation (6):

$$q\mu \exp[-\mu(t-\alpha)][\mu(t-\alpha)]^{L/1}/(L-1)!$$

$$h_\theta(t) = \begin{cases} +q \sum_{n=2}^{\infty} p^{n-1} \exp\{-\mu[t - n\alpha - (n-1)\beta]\} \mu^{nL} & \\ [t - n\alpha - (n-1)\beta]^{nL-1}/(nL-1)! & t > \alpha \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

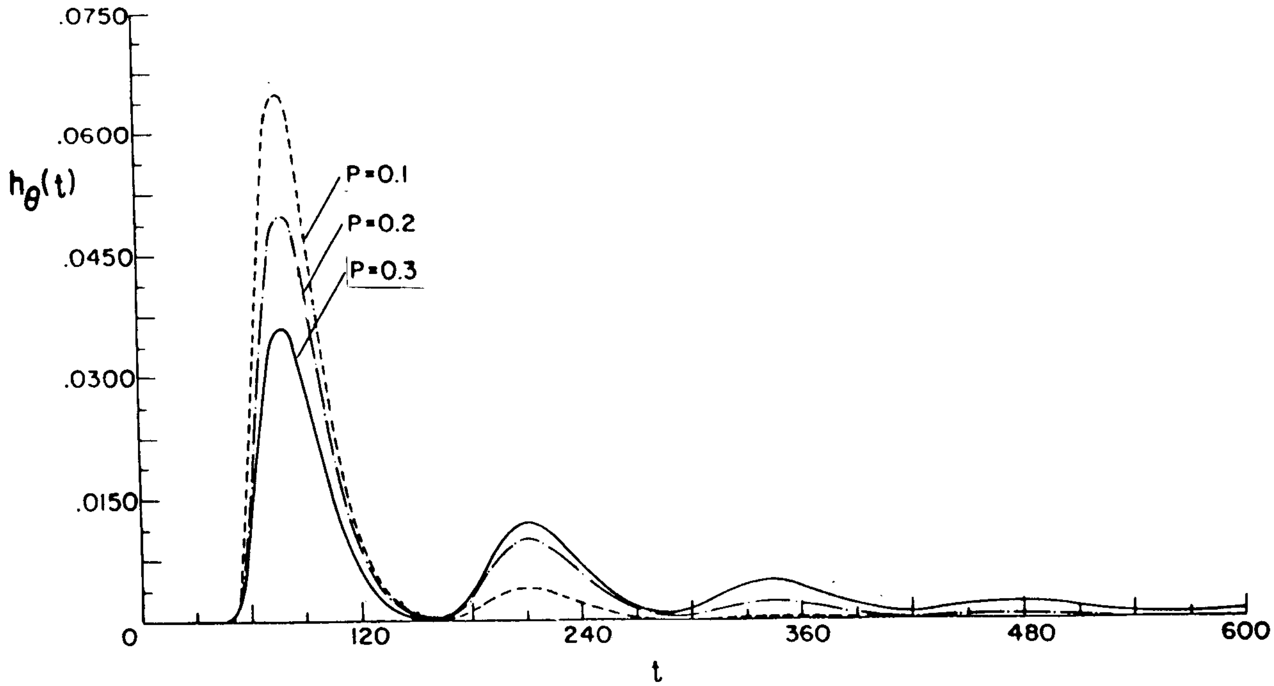

Fig. 3. Total cell time distribution for p = 0.1, 0.2 and 0.3.

The mean and the variance of the automated assembly time for one *good* assembly are:

$$\mu_\theta = q^{-1}(\alpha + L/\mu + p\beta) \tag{25}$$

and

$$\sigma_\theta^2 = p\,q^{-2}(\alpha + L/\mu + \beta)^2 + q^{-1}(L/\mu)^2 . \tag{26}$$

Numerical example: Suppose that α = 21sec., β = 135sec., L = 4 and $1/\mu$ = 30sec. The p.d.f. $h_\theta(t)$ for p = 0.1, 0.2 and 0.3 are illustrated in Fig. 3.

The oscillations on the curves represent completions after 0. 2, 3, 4, . . . reworks. The frequency of these damped oscillations is $1/(\mu_M + \mu_R)$. As p increased the spikes become more pronounced. Knowledge of this stochastic phenomenon is essential for interpreting operational (or simulated) data and in designing the material flow interface between unitary cells and their adjacent facilities. For example, it has been shown that such random variations may degrade the average plant performance in terms of schedule effectiveness and work-in-progress levels [18].

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